

Mathematics

4 Points — First Questionnaire

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

Probability — 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus

of Polynomials, Rational Functions and

Root Functions

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

(3) For draft paper, use pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of your exam.

מתימטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות

20×2 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

20×1 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

של פולינומים, של פונקציות

רציונליות ושל פונקציות שורש

20×2 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש

באפשרויות התכנות במחשבון הניתן

לתכנות. שימוש במחשבון גרפי או

באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום

במחברת את שלבי הפתרון, גם כאשר

החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או

לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry and Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

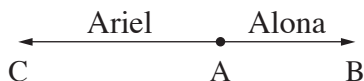
1. Alona and Ariel left City A at the same time, each in his/her own car.

Alona traveled from City A to City B, while

Ariel traveled from City A to City C.

The distance between City A and City B

is 60 km.



Alona traveled at a speed that was 1.5 times higher than Ariel's speed.

They both traveled at a constant speed for the entire journey.

When Alona reached City B, Ariel had traveled 40% of the distance between City A and City C.

- א. What is the distance between City A and City C?
ב. Ariel reached City C one hour after Alona reached City B.
At what speed did Ariel travel?

2. Points B $(-2, 1)$ and C $(10, -5)$ are located on a circle whose center is M $(3, -4)$.

Two segments that are tangent to the circle at Points B and C are drawn from Point A, which is outside the circle, as shown in the figure.

- א. (1) Find the equations of the lines AB and AC.

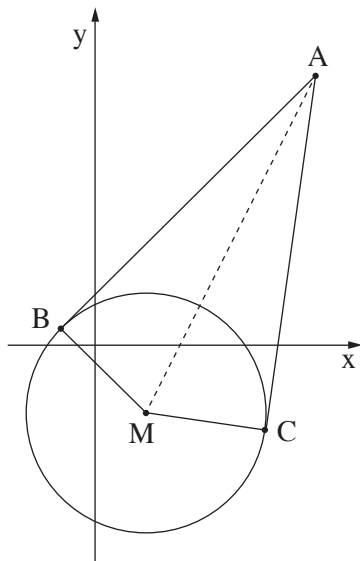
- (2) Find the coordinates of Point A.

- ב. (1) Find the length of Segment AM.

- (2) Find the equation of the circle that circumscribes [חוסם] Triangle ABM.

- (3) Is Point C located on the circle whose equation you found?

Explain your answer.



3. In a game of chance, each player throws a single dice twice. The dice is a balanced, traditional dice [קוביית משחק הוגנת].

Each time the dice is thrown, if the number shown is 3, the player receives 5 points, if the number is greater than 3, the player receives 10 points, and if the number is smaller than 3, the player receives no points.

- א. What is the probability that a player will accrue at least 15 points?

- ב. We know that one of the players accrued at least 15 points. What is the probability that the number on the dice was greater than 3, both times he threw the dice?

- ג. Four players are participating in the game.

What is the probability that exactly two of them will each accrue at least 15 points?

Part Two — Planar Geometry and Planar Trigonometry

(20 points)

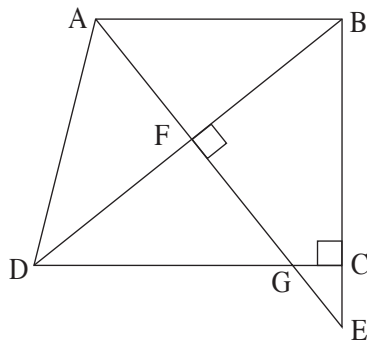
Answer one of the questions 4-5.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. ABCD is a right trapezoid [טרפז ישר זווית] ($AB \parallel DC$, $\angle BCD = 90^\circ$).

E is a point on the extension of Side BC, such that Segment AE is perpendicular [מאונך] to Diagonal BD and intersects it at Point F.

AE intersects Segment DC at Point G, as shown in the figure.



א. Prove that $\angle AEB = \angle BDC$.

Given: $DC = BE$.

ב. Prove that $\triangle DCB \cong \triangle EBA$.

Given: $CB = 4CE$.

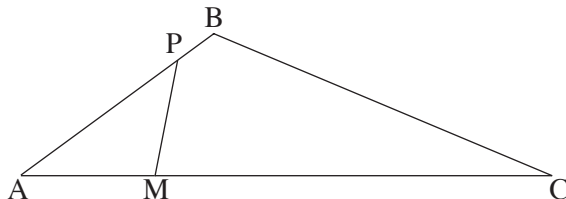
ג. (1) Prove that $\triangle GCE \sim \triangle ABE$.

(2) Find the ratio $\frac{GC}{AB}$.

5. In Triangle ABC, Point P is located on Side AB,
 and Point M is located on
 Side AC (see figure).

Let x denote AP.

Given:



$$PM = 0.6x,$$

$$\angle ABC = 120^\circ, \angle AMP = 100^\circ,$$

$$AM = 4 \text{ cm}, \quad MC = 12 \text{ cm}.$$

- א. (1) Calculate the angle PAM.
- (2) Calculate the length of Side BC.
- ב. Calculate the length of Segment BM.
- ג. Find the ratio of the areas of the triangles $\frac{S_{\triangle AMB}}{S_{\triangle BMC}}$. Explain your answer.

Part Three — Differential and Integral Calculus of Polynomials, Rational Functions and Root Functions

(40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x) = \frac{2x^2 + 4}{x^2 - a}$; $a > 0$ is a parameter.

Answer Item א. Write your answers in terms of a , if necessary.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
(2) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes (if there are such points).
(3) Find the horizontal asymptote of the function $f(x)$.

The function $f(x)$ has a vertical asymptote $x = 1$.

- ב. Find a .

Insert the value of a that you found for Item ב and answer Items ג, ד and ה.

- ג. (1) Does the function $f(x)$ have an additional vertical asymptote?
If so, what is it? If not, explain.
(2) Find the coordinates of the extremum [נקודת הקיצון] of the function $f(x)$ and determine what type of extremum it is.
(3) Find the intervals on which the function $f(x)$ increases and decreases.
- ד. Sketch a graph of the function $f(x)$.
- ה. For which values of k is there no solution to the equation $f(x) = k$?
Explain.

7. Given: The function $f(x) = \frac{4}{\sqrt{x+16}}$.

- א. (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the coordinates of the points at which the graph of the function $f(x)$ intersects the axes (if there are such points).
- (3) Find the vertical asymptote of the function $f(x)$.
- (4) Find the intervals on which the function $f(x)$ increases and decreases (if there are such intervals).
- (5) Sketch the graph of the function $f(x)$ over the interval $x \leq 0$.

Given: The function $g(x) = f(x) - 2$.

- ב. (1) Find the coordinates of the points at which the graph of the function $g(x)$ intersects the axes.
- (2) Sketch the graph of the function $g(x)$ over the interval $x \leq 0$.
- ג. Find the area of the region bounded by the graph of the function $g(x)$ and the axes.

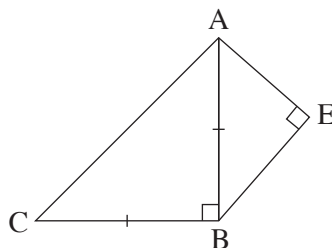
8. ABC is a right isosceles triangle [משולש ישר זווית ושווה שוקיים] ($\angle ABC = 90^\circ$).

A right triangle AEB was constructed on Side AB, such that AB is the hypotenuse [יתר] of Triangle AEB, as shown in the figure.

Given: The sum of the lengths of the legs [ניצבים] of Triangle AEB is 6 cm.

Let x denote the length of Side AE.

- א. Express the area of Triangle ABC in terms of x .
- ב. For what value of x will the area of the quadrilateral AEBC be smallest [מינימלי]?



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך