

**The State of Israel
Ministry of Education**

Exam Type:

א. *Bagrut* for high schools

ב. *Bagrut* for external examinees

Exam Date: Summer 2016, *Moed Bet*

Exam Number: 035806, 316

Appendix: Formula sheets for 5 points

English Translation (3)

מדינת ישראל

משרד החינוך

סוג הבחינה:

א. בגרות לבתי ספר על-יסודיים

ב. בגרות לנבחנים אקסטריניים

מועד הבחינה: קיץ תשע"ו, **מועד ב**

מספר השאלון: 316, 035806

נספח: דפי נוסחאות ל-5 יחידות לימוד

תרגום לאנגלית (3)

Mathematics
5 Points — First Questionnaire

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra and Probability

$$2 \times 20 = 40 \text{ points}$$

Part Two: Planar Geometry and Planar Trigonometry

$$1 \times 20 = 20 \text{ points}$$

Part Three: Differential and Integral Calculus

$$2 \times 20 = 40 \text{ points}$$

$$\text{Total} = 100 \text{ points}$$

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of the exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language / foreign language-Hebrew dictionary

ד. Special instructions:

(1) Do not copy the question; only indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of the exam.

(3) For draft paper, use the pages in the test booklet or paper you receive from the proctor.

Use of other draft paper may lead to the disqualification of the exam.

Good Luck!

מתמטיקה

5 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה והסתברות

$$2 \times 20 = 40 \text{ נקודות}$$

פרק שני: גאומטריה וטריגונומטריה במישור

$$1 \times 20 = 20 \text{ נקודות}$$

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

$$2 \times 20 = 40 \text{ נקודות}$$

$$\text{סה"כ} = 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש

במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום במחברת

את שלבי הפתרון, גם כאשר החישובים

מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מן המושגי.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Two technicians, Gal and Danny, were working at assembling computers. They each worked at a constant rate.
 - א. On the first day of work, both technicians assembled the same number of computers.
Gal started working at 8:00 and finished at 15:00.
Danny started working after 8:00 and before 9:00 and finished working at 13:00.
We know that Gal and Danny assembled the same number of computers between the time they each started working and 9:00.
How long after 8:00 did Danny start working?
 - ב. On the second day of work, Gal and Danny began working at the same time and finished working at the same time.
That day, together, they assembled the same total number of computers as they had assembled together on the first day.
For how long did the technicians work on the second day?

2. Given: an arithmetic sequence that has n terms. The common difference [הפרש הסדרה] of the given sequence is 3.
- א. An additional term was inserted between every two consecutive terms, to form a new arithmetic sequence.
- (1) Show that the ratio between the sum of the terms in the new sequence and the sum of the terms in the given sequence is $\frac{2n-1}{n}$.
- (2) Given: The ratio that appears in Sub-item (1) is equal to 1.9.
The sum of all the terms inserted into the given sequence is 130.5.
Find the first term in the given sequence.
- ב. Another arithmetic sequence was formed by inserting k terms between every two consecutive terms of the given sequence. Express the common difference of the obtained sequence in terms of k .
3. Chess is a two-player game that can end in a win for one of the players or in a tie.
- Yael and Anna are playing against each other in a chess tournament which has two rounds.
- The probability of any one player winning any individual match is constant over the whole tournament.
- א. The first round consists of 4 matches. The probability that Yael will win 2 matches or 3 matches is 10 times greater than the probability that Yael will win 4 matches.
- Calculate the probability that Yael will win any individual match.
- The second round consists of 2 matches.
- The probability that the second round will end in a tie is 0.34.
- ב. What is the probability that Anna will win any individual match?
- ג. Calculate the probability that Anna will win the second match, if we know that this round will end in a tie.

Part Two — Planar Geometry and Planar Trigonometry

(20 points)

Answer one of the questions 4-5.

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

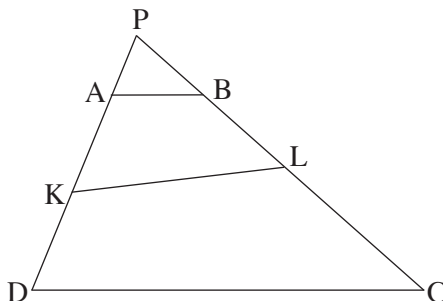
4. Given: Triangle PDC.

Points B and L are located on Side PC.

Points A and K are located on Side PD,
 as shown in the figure.

Given: Quadrilateral ABLK can be circumscribed [בר-חסימה] by a circle and Quadrilateral KLCD can also be circumscribed by a circle.

א. Prove that $AB \parallel DC$.



Given: $PA = 3$ cm, $PB = 4$ cm,

the area of Triangle ABP is S cm²,

the area of Quadrilateral ABCD is $24S$ cm².

ב. Can Quadrilateral ABCD be circumscribed by a circle? Explain.

ג. Find the length of Side PD.

ד. Also given: $BL = 5$ cm.

Use the similarity of the triangles [דמיון משולשים] to express the area of Quadrilateral KLCD in terms of S .

5. Trapezoid ABCD ($AB \parallel DC$) is circumscribed by a circle.

The center, O, of the circle is located within the trapezoid (see figure).

The radius of the circle is R and the height of the trapezoid is h .

Given: $\angle COD = \alpha$, $\angle BOA = 3\alpha$.

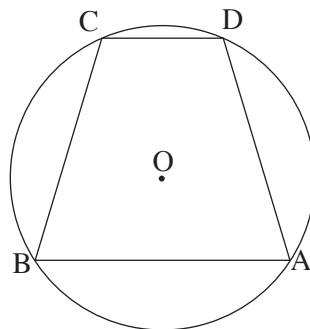
א. Express $\angle DAB$ in terms of α .

ב. Express the length of a leg [שוק] of the trapezoid in terms of α and R .

ג. Express the length of a leg of the trapezoid in terms of α and h .

ד. Given: The area of Triangle COD is

$$\frac{h^2}{12 \cos^2 \frac{\alpha}{2}}. \text{ Find } \alpha.$$



/Continued on page 5/

Part Three — Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions and Trigonometric Functions

(40 points)

Answer two of the questions 6-8 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x) = \frac{2\cos^2 x - 1}{2\cos^2 x}$.

א. For the interval $0 \leq x \leq \frac{\pi}{2}$:

- (1) Find the domain [תחום ההגדרה] of the function $f(x)$.
- (2) Find the asymptotes of the function $f(x)$ that are perpendicular to the x -axis (if there are any).
- (3) Find the points at which the function $f(x)$ intersects the x -axis (if there are any).
- (4) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$ (if there are any) and determine what type of extrema they are.

ב. For the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$:

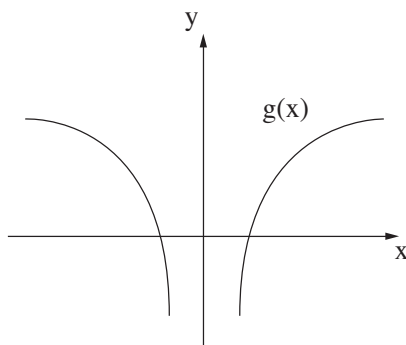
- (1) Show that the function $f(x)$ is even [זוגית].
- (2) Sketch the graph of the function $f(x)$.

ג. Find the area of the region in the first quadrant that is bounded by the graph of the function $f(x)$, the x -axis and the y -axis.

7. The figure to the right describes the graph of the function $g(x)$.

The functions $g(x)$, $g'(x)$ and $g''(x)$ are defined for every value of x that is different from 0 and have no extrema [נקודות קיצון] or inflection points [נקודות פיתול].

The line $x = 0$ is the vertical asymptote for the graphs of each of these functions.



- א. (1) Sketch a graph of the derivative function $g'(x)$. Explain your reasoning.
 (2) Sketch a graph of the second-derivative function $g''(x)$. Explain your reasoning.

Given: The area of the region bounded by the graph of the second-derivative function $g''(x)$, the x -axis and the lines $x = 1$ and $x = 2$ is equal to 5.25.

- ב. The line $x = 1$ intersects the graph of the derivative function $g'(x)$ at Point A, and the line $x = 2$ intersects this graph at Point B. Find the difference between the y -coordinate of Point A and the y -coordinate of Point B. Explain.
- ג. The expression $y = \frac{a}{x^3}$ describes one of the functions $g(x)$, $g'(x)$ or $g''(x)$.
 a is a parameter that is greater than 0.
 (1) Determine which of the functions the expression describes. Explain your answer.
 (2) Find the value of a .

8. In the right triangle [משולש ישר זווית] ABC ($\angle ABC = 90^\circ$), the length of the hypotenuse [יתר] is k cm (k is a parameter).

The side AB is also the hypotenuse of Triangle ADB , which is a right isosceles [שווה שוקיים וישר זווית] triangle ($\angle ADB = 90^\circ$).

- א. Let AB equal x and express BC in terms of x and k .
- ב. Given: The maximal value of the product [מכפלה] $BC \cdot AD^2$ is $3\sqrt{3}$. Find the area of Triangle ADB (a numerical value), when the product $BC \cdot AD^2$ is greatest (maximal).

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך