

Mathematics

4 Points — First Questionnaire

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

Probability — 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of the exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language / foreign language-Hebrew dictionary

ד. Special instructions:

(1) Do not copy the question; only indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of the exam.

(3) For draft paper, use the pages in the test booklet or paper you receive from the proctors.

Use of other draft paper may lead to the disqualification of the exam.

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות

20×2 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

20×1 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

20×2 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות

התכנות במחשבון הניתן לתכנות. שימוש

במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.
 Lack of detail may lower your score or lead to the disqualification of the exam.

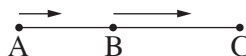
Part One — Algebra, Analytical Geometry and Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Yoav was riding his bike. He left City A, passed through City B and arrived at City C.

The distance from B to C is 40 km greater than the distance from A to B.



Yoav cycled from B to C at a constant speed that was 20% greater than the constant speed at which he cycled from A to B.

The period of time that Yoav spent cycling from B to C was 1.25 times longer than the time he spent cycling from A to B.

If Yoav had cycled from B to C at the speed at which he cycled from A to B, the trip from B to C would have taken him 6 hours.

- א. Find the speed at which Yoav cycled from A to B.
- ב. Find the distance AB.

2. The diagonals of the square ABCD intersect at Point M (see figure).

The coordinates of Vertex A are (5, 5).

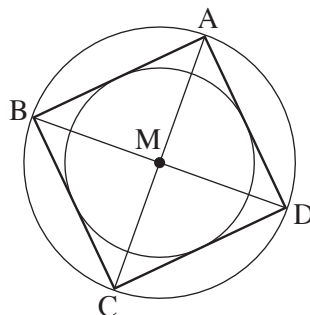
The equation of Diagonal BD is $y = -\frac{1}{3}x$.

- א. Find the equation of Diagonal AC.

- ב. Find the equation of the circle that circumscribes [חוסם] the square.

- ג. Calculate the length of a side of the square.

- ד. Calculate the length of the radius of the circle that is circumscribed [חוסם] by the square (see figure).



3. Shahar bought a box of tennis balls that contained balls of two colors:
4 yellow balls and 6 green balls. Shahar removed 3 random balls from the
box, one after the other (without returning them).
- א. (1) What is the probability that Shahar removed 3 yellow balls?
(2) What is the probability that Shahar removed 3 balls of the
same color?
- ב. Dana bought 3 boxes of tennis balls. Each of the boxes that she bought
was identical to the box that Shahar bought.
Dana removed one random ball from each of the boxes.
- (1) What is the probability that Dana removed 3 yellow balls?
(2) What is the probability that Dana removed at least one green
ball?

Part Two — Planar Geometry and Planar Trigonometry

(20 points)

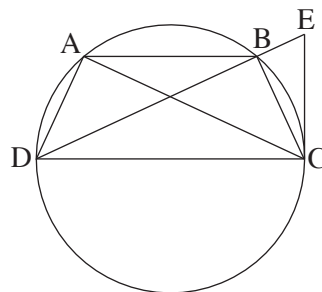
Answer one of the questions 4-5.

Note! If you answer more than one question, only the first answer in your answer booklet will be graded.

4. An isosceles trapezoid [טרפז שווה-שוקיים], ABCD, is circumscribed by a circle.

The tangent to the circle that passes through Point C intersects the extension of Diagonal DB at Point E.

CD is the diameter of the circle (see figure).

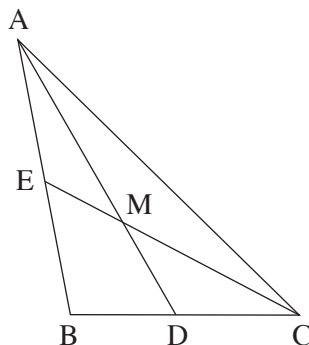


- א. Prove that $\triangle DAC \sim \triangle ECD$.
- ב. Given: $AC = 25$ cm , $DE = 36$ cm.
Calculate the radius of the circle.
- ג. Calculate the area of Triangle DAC.

5. AD and CE are medians [תיכונים] of Triangle ABC that intersect at Point M (see figure).

Given: $AD = 12$ cm , $CE = 9$ cm , $\angle CMD = 40^\circ$.

- א. Calculate the lengths of Segments MC and MD.
- ב. Calculate the length of Side BC.
- ג. Calculate the size of Angle $\angle MCD$.
- ד. Calculate the area of Triangle ADB.



Part Three — Differential and Integral Calculus of Polynomials, Rational Functions and Root Functions

(40 points)

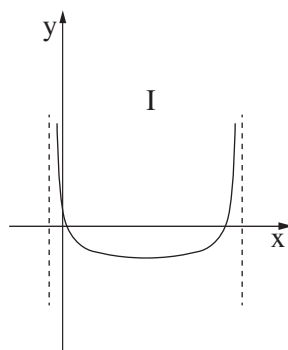
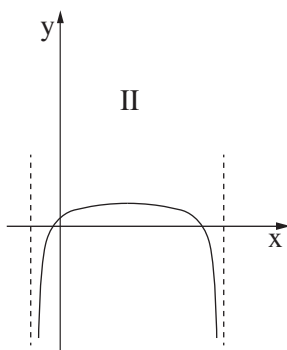
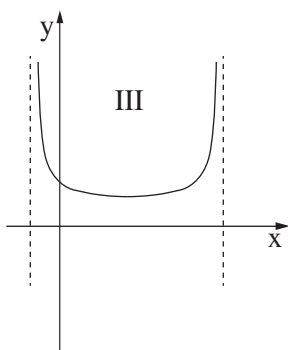
Answer two of the questions 6-8 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x) = \sqrt{-x^2 + 6x + 7}$

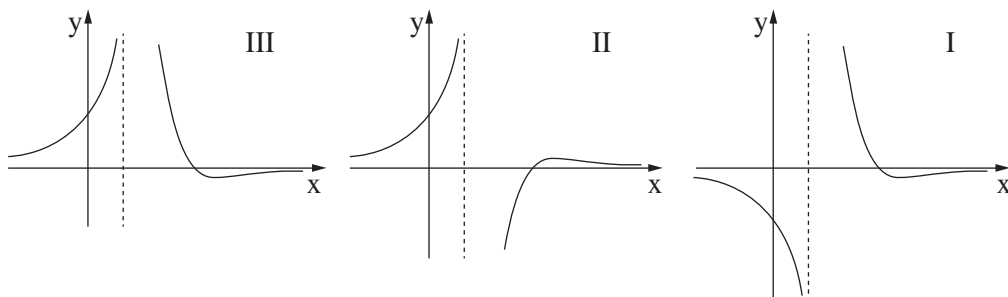
- א. Find the domain [תחום ההגדרה] of the function.
- ב. Find the absolute extrema [נקודות הקיצון המוחלט] of the function and determine what type of extrema they are.
- ג. Sketch the graph of the function.
- ד. Given: the function $g(x)$ for which $g(x) = \frac{1}{f(x)}$
 - (1) Find the domain of the function $g(x)$.
 - (2) Which one of the graphs below, I, II or III, represents the graph of the function $g(x)$?

Explain.



7. Given: The function $f(x) = \frac{2-x}{(x-1)^2}$.
- Find the domain [תחום ההגדרה] of the function.
 - Find the points at which the graph of the function intersects the axes.
 - Find the asymptotes of the graph of the function that are perpendicular to the axes.
 - Find the intervals over which the function increases and decreases.
 - Sketch the graph of the function.
 - Three graphs, I, II and III, are shown below.

Which of the graphs represents the derivative function $f'(x)$?



8. The figure to the right shows the graphs of two functions:

$$f(x) = x^2 - 6x + 5$$

$$g(x) = x^2 - 10x + a$$

a is a parameter.

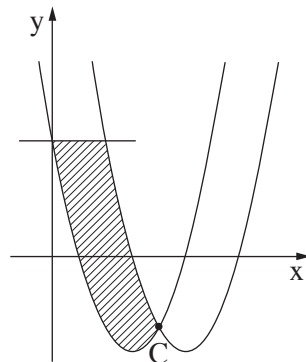
The graphs intersect at Point C (see figure).

The x-coordinate of Point C is 4.

- Find the value of a .
- A line was drawn through the point at which one of the graphs intersects the y-axis.

This line is parallel to the x-axis, as shown in the figure.

Find the area of the region that is bounded by the graphs of the two functions and the line that is parallel to the x-axis (the shaded area in the figure).



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך