

Mathematics

5 Points — First Questionnaire

Instructions for examinees

- א. Duration of the exam: Three and a half hours
- ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra and Probability
– $2 \times 20 = 40$ points
Part Two: Planar Geometry and Planar Trigonometry
– $1 \times 20 = 20$ points
Part Three: Differential and Integral Calculus
– $2 \times 20 = 40$ points
Total – 100 points
- ג. Material that may be used during the exam:
 - (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of the exam.
 - (2) Formula sheets (attached).
 - (3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.
- ד. Special instructions:
 - (1) Do not copy the question, but do indicate the number of the question you are answering.
 - (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of the exam.
 - (3) For draft paper, use pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of the exam.

Good Luck!

מתמטיקה

5 יחידות לימוד - שאלון ראשון

הוראות לנבחן

- א. משך הבחינה: שלוש שעות וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה והסתברות
 $20 \times 2 = 40$ נקודות
פרק שני: גאומטריה וטריגונומטריה
במישור $20 \times 1 = 20$ נקודות
פרק שלישי: חשבון דיפרנציאלי
ואינטגרלי $20 \times 2 = 40$ נקודות
סה"כ – 100 נקודות
- ג. חומר עזר מותר בשימוש:
 - (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
 - (2) דפי נוסחאות (מצורפים).
 - (3) מילון עברי-לועזי/לועזי-עברי.
- ד. הוראות מיוחדות:
 - (1) אל תעתיק את השאלה; סמן את מספרה בלבד.
 - (2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
 - (3) לטיטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים. שימוש בטיטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail. Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra and Probability (40 points)

Answer two of the questions 1-3 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. Two cars left City א at the same time, both traveling toward City ב.
The distance between the two cities is 300 km.
The first car traveled at a speed that was 25 kph faster than the speed of the second car.
1.5 hours after the cars left City א, the first car slowed down, reducing its speed by half. That car reached City ב $\frac{1}{2}$ an hour after the second car.
א. Find the speed of the second car, if we know that it was greater than 60 kph.
ב. At how many hours after the moment of departure from City א (before the second car caught up with the first car) was there a distance of 12.5 km between the two cars?
(Find both options.)
2. Given: an arithmetic sequence a_n for which $a_4 + a_8 + a_{12} + a_{16} = 224$
א. Find the sum of the first 19 terms in Sequence a_n .
The sequence S_n is the series of partial sums of Sequence a_n :
 $S_1, S_2, S_3, \dots$
Also given: For every natural n , $S_n = n \cdot a_n$.
ב. Show that the common difference [הפרט] of the sequence a_n is 0.
ג. Using the previous items, find a_1 .
Given: the sequence b_n for which $b_{n+1} - b_n = a_n + S_n$, for every natural n
ד. Using the previous items for this question, find the sum
 $(b_2 - b_1) + (b_3 - b_2) + (b_4 - b_3) + \dots + (b_{20} - b_{19})$.



3. 20% of the people who took a college entrance exam were from kibbutzim. 40% of the people who took the exam were from moshavim and 40% were from cities.

70% of the people who took the exam passed.

$\frac{1}{8}$ of the examinees from moshavim failed the exam.

The probability of randomly choosing, from among all of the people who took the exam, a person from a city who passed the test is 2.5 times greater than the probability of randomly choosing a person from a kibbutz who passed the exam.

- א. What is the probability of randomly choosing a person who is not from a city, from among all of the people who failed the exam?

- ב. (1) Moshe passed the exam.

What is the probability that he is not from a moshav?

- (2) Five students passed the exam.

What is the probability that at least one of them is from a moshav?

Part Two — Planar Geometry and Planar Trigonometry

(20 points)

Answer one of the questions 4-5.

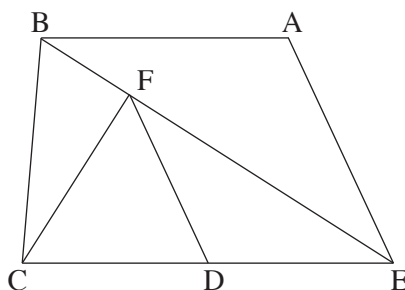
Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

4. Given: the trapezoid ABCE ($AB \parallel EC$)
Point F is located on the diagonal BE such that $CF \perp BE$.

Point D is the midpoint of the base CE
(see figure).

Given: $\angle CEB = \angle AEB$

$$ED = 3a, \quad EA = 4a.$$



- א. Prove that $\triangle EAB \sim \triangle EDF$.
- ב. Given: The area of the triangle EAB is S.
Express the area of the triangle CEF in terms of S.
- ג. The continuation of DF intersects AB at Point G.
Express the area of Triangle BFG in terms of S.

5. Given: the isosceles triangle [משולש שווה-שוקיים] ABC ($AB = AC$)

AE is the height to the base BC and BT is the median [תיכון] to Side AC (see figure).

Given: $BC = 2k$, $\angle TBC = \alpha$, $\angle ACB = \beta$.

- א. (1) Express the length of TC in terms of k and β only.

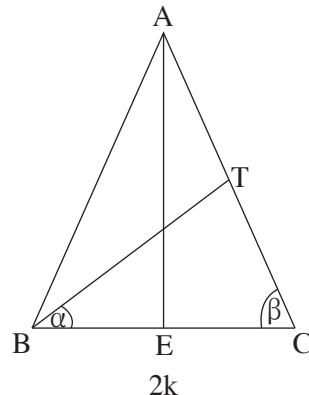
- (2) Using Sub-item א(1), show that

$$\sin(\alpha + \beta) = 4 \sin \alpha \cdot \cos \beta .$$

- ב. Also given: $TE = 5$ cm and $k = 4$ cm.

- (1) Find β .

- (2) Find α .



Part Three — Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions and Trigonometric Functions (40 points)

Answer two of the questions 6-8 (each question – 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x) = x^2 - \sin(2x)$ over the interval $-\frac{\pi}{2} \leq x \leq 0$.
Answer the items below for the given interval.

- א. Find the greatest slope and the smallest slope of the graph of the function $f(x)$.

- ב. Sketch the graph of the derivative function $f'(x)$.

- ג. (1) Find the intervals over which the graph of the function $f(x)$ curves upward, $\cup$, and the intervals over which it curves downward, $\cap$.
(2) Sketch the graph of the function $f(x)$.

7. Given: The function $f(x) = \frac{ax^3 + 2ax}{\sqrt{x^4 + 4x^2 + 4}}$.

a is a parameter greater than 0.

- א. Find the domain [תחום ההגדרה] of the function $f(x)$.
- ב. Is the function $f(x)$ even or odd? Explain.
- ג. The region bounded by the graph of the function $f(x)$, the x -axis and the lines $x = 1$ and $x = -1$ has an area of 4.

Find the value of a .

- ד. Given: For the function $g(x)$, $f(x) = g'(x)$.

One of the points at which the graph of the function $f(x)$ intersects the graph of the function $g(x)$ is a point at which $x = 0$.

- (1) Show that for the function $g(x)$: $g(x) = 2x^2$.
- (2) Find the interval over which $f(x) > g(x)$.

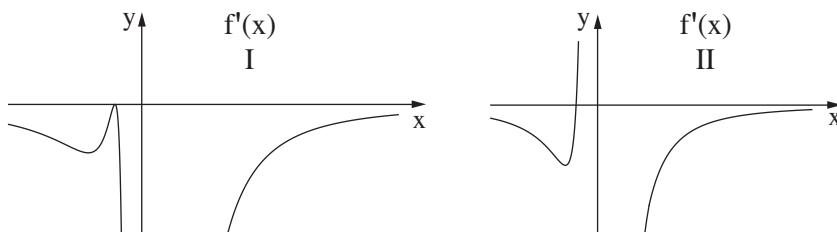
8. Given: The function $f(x) = \left(1 + \frac{1}{x}\right)^n$; $x \neq 0$.

n is a natural number greater than 1.

א. Find the asymptotes of the function $f(x)$ that are perpendicular [מאונכות] to the axes.

ב. Show that for an odd value of n , $f'(x) \leq 0$ whenever $x \neq 0$.

Two graphs are presented below, I and II. (The graphs show all of the extrema [נקודות הקיצון].)



One of the graphs represents a sketch of the derivative function $f'(x)$ for an even value of n and the other graph represents a sketch of the derivative function $f'(x)$ for an odd value of n .

Use Graphs I and II to answer Items ג, ד and ה.

ג. For an odd value of n :

- (1) How many extrema does the function $f(x)$ have (if any)? Explain.
- (2) How many inflection points [נקודות פיתול] does the function $f(x)$ have? Explain.

ד. For an even value of n :

- (1) How many extrema does the function $f(x)$ have (if any)? Explain.
- (2) How many inflection points [נקודות פיתול] does the function $f(x)$ have? Explain.

(3) Sketch the graph of the function $f(x)$.

ה. Given functions: $g(x) = \left(1 + \frac{1}{x}\right)^3$ and $h(x) = \left(1 + \frac{1}{x}\right)^4$.

What is the sign of the product [מכפלה] $g''(x) \cdot h''(x)$ when $x > 0$?

Explain.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך