

Mathematics

4 Points — First Questionnaire

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

Probability — 2×20 — 40 points

Part Two: Planar Geometry and Planar Trigonometry

— 1×20 — 20 points

Part Three: Differential and Integral Calculus

— 2×20 — 40 points

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of the exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of the exam.

(3) For draft paper, use pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of the exam.

Good Luck!

מתמטיקה

4 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות

20×2 — 40 נקודות

פרק שני: גאומטריה וטריגונומטריה במישור

20×1 — 20 נקודות

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

20×2 — 40 נקודות

סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש

באפשרויות התכנות במחשבון הניתן

לתכנות. שימוש במחשבון גרפי או

באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום

במחברת את שלבי הפתרון, גם כאשר

החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטיטוה יש להשתמש במחברת הבחינה או

בדפים שקיבלת מהמשגיחים.

שימוש בטיטוה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

Part One —

Algebra, Analytical Geometry and Probability (40 points)

Answer two of the questions 1-3 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. An importer purchased a number of identical computers at the cost of 1200 shekels per computer.
He sold the computers to a store, charging the same price for each computer, and made a certain percent profit on each computer.
The store sold each computer for 1728 shekels and made the same percent profit on each computer as the importer had made.
 - א. What percent profit did the importer make?
 - ב. Yossi bought a computer directly from the importer for a price that was 42% higher than the price that the importer had paid.
Did Yossi pay less for his computer than someone who bought an identical computer from the store? Explain.

2. Given: two circles, I and II:

I. $x^2 + y^2 = 36$

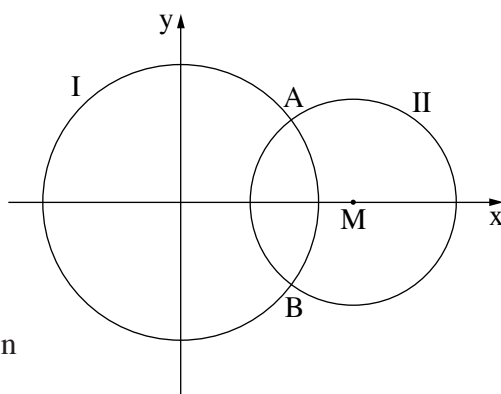
II. $(x - 7.5)^2 + y^2 = 20.25$

The circles intersect at

Points A and B.

A is located in the first quadrant (see figure).

- א. Find the coordinates of Points A and B.
- ב. A tangent to Circle II was drawn through Point A.
Find the equation of this tangent.
- ג. The tangent you found in Item ב intersects Circle I at an additional point, C. Find the area of the triangle ACM. M – the center of Circle II.



Continued on page 3 ►

3. To be accepted into the computer science program at a university, applicants must pass an entrance exam.

Many high school graduates took the exam; some had studied computer science in high school and some had not.

The percentage of examinees who had studied computers in high school was 3 times greater than the percentage of examinees who had not studied computers.

The percentage of examinees who passed the exam was 4 times greater than the percentage who failed it.

The percentage of examinees who passed the exam and had also studied computers was 65%.

- א. If we randomly choose a person from among all those who took the exam, what is the probability that that person did not study computers and did pass the exam?
- ב. We know that a certain person passed the exam.
What is the probability that he did not study computers in high school?
- ג. Two people are randomly chosen from among all those who took the exam.
What is the probability that no more than one of them passed the exam?

Part Two — Planar Geometry and Planar Trigonometry (20 points)

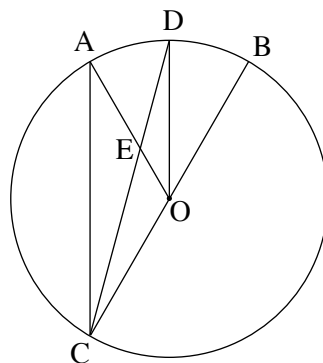
Answer one of the questions 4-5.

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

4. BC is the diameter of a circle whose center is O.
The chord [מיתר] CD intersects the radius AO at Point E. Point D is the midpoint of the arc [קשת] AB (see figure).

Let α represent the angle ACD ($\angle ACD = \alpha$).

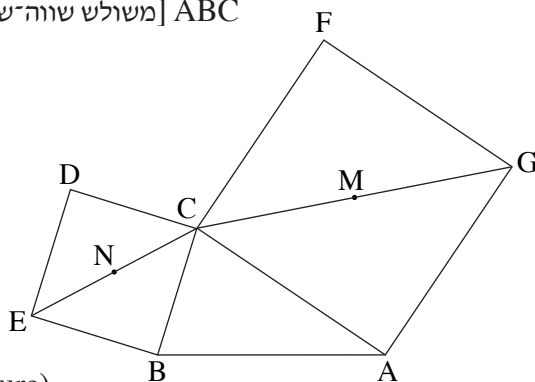
- א. (1) Prove that $\angle ACO = \angle AOD$.
(2) Prove that $AC \parallel DO$.
- ב. (1) Express the size of the angle DAO in terms of α .
(2) For the quadrilateral ACOD to be a parallelogram [מקבילית], what must the value of α be?
Explain.



5. Given: an isosceles triangle [משולש שווה-שוקיים] ABC
 ($AB = AC$).

A square, $ACFG$, whose
 diagonals intersect at Point M ,
 was added to Side AC .

A square, $BCDE$, whose
 diagonals intersect at Point N ,
 was added to the base BC (see figure).



Given: $AB = AC = 6$ cm.

$BC = 4$ cm.

- א. Find the length of the diagonal of Square $ACFG$ and the length of the diagonal of Square $BCDE$.
- ב. Find the size of the base angle of Triangle ABC .
- ג. Show that the area of Triangle BCM is equal to the area of Triangle ABN .
- ד. Find the length of the segment AN .

Part Three — Differential and Integral Calculus of Polynomials, Rational Functions and Square-Root Functions (40 points)

Answer two of the questions 6-8 (each question — 20 points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

6. Given: The function $f(x) = \frac{m - 4x}{(x - 1)^2}$; m is a parameter.

The function $f(x)$ has an extremum [נקודת קיצון] at a point at which $x = 3$.

א. Find the domain [תחום ההגדרה] of the function $f(x)$.

ב. Find the value of the parameter m .

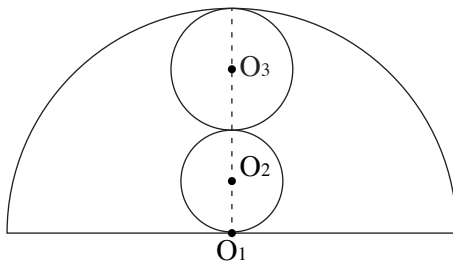
Insert the value $m = 8$ and answer Items ג, ד and ה.

- ג. (1) Find the asymptotes of the function $f(x)$ that are parallel to the axes.
- (2) Find the points at which the graph of the function $f(x)$ intersects the axes.
- (3) Find the coordinates of the extrema [נקודות הקיצון] of the function $f(x)$ (if there are any) and determine what type of extrema they are.
- (4) Find the intervals on which the function $f(x)$ increases and decreases.
- ד. Sketch a graph of the function $f(x)$.
- ה. Use the graph that you sketched to determine for which values of x the following statements are both true: $f(x) > 0$ and $f'(x) > 0$.

7. The derivative of the function $f(x)$ is $f'(x) = 3x^2 - 12x + 9$.
- א. (1) Find the x -coordinates of the extrema [נקודות קיצון] of the function $f(x)$ and determine what type of extrema they are.
- (2) The line $y = 4$ is a tangent to the graph of the function $f(x)$ at the maximum point of the function. Find the function $f(x)$.
- ב. (1) Find the points at which the graph of the function $f(x)$ intersects the axes.
- (2) Sketch the graph of the function $f(x)$.
- ג. A line that is perpendicular to the x -axis was drawn through the minimum point of the function $f(x)$.
- Find the area of the region bounded by the graph of the function $f(x)$, the tangent $y = 4$, the y -axis and the perpendicular line.

8. Two circles whose centers are O_2 and O_3 are circumscribed by a semicircle whose center is O_1 and whose radius is 10 cm.

The circles are tangent to each other, as shown in the figure. (The three centers are located on one line.)



- א. For what length of the radius of the circle with center O_2 and what length of the radius of the circle with center O_3 will the total area of Circles O_2 and O_3 be smallest [מינימלי]?
- ב. When the total area of Circles O_2 and O_3 is smallest, what is the sum of the circumferences [היקפים] of these circles?

Given: Area of a circle $= \pi R^2$.

Circumference of a circle $= 2\pi R$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך