

Mathematics

3 Points — Third Questionnaire

Instructions for examinees

א. Duration of the exam: Two hours

ב. Exam structure and breakdown of points:

This exam has six questions in the topic areas:
Algebra and Differential and Integral Calculus.

You must answer four questions –

$$4 \times 25 = 100 \text{ points}$$

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or the programming options may lead to the disqualification of the exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/
foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of the exam.

(3) For draft paper, use the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of the exam.

מתמטיקה

3 יחידות לימוד — שאלון שלישי

הוראות לנבחן

א. משך הבחינה: שתיים.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

עליך לענות על ארבע שאלות –

$$100 = 25 \times 4 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום במחברת

את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note! Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer four of the questions 1-6 (each question – 25 points).

Note! If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. A school principal wants to buy 80 pieces of school equipment. The purchase she wants to make consists of computers and interactive whiteboards.

Each computer costs 1200 shekels, and each interactive whiteboard costs 2000 shekels.

The entire purchase will cost 144,000 shekels.

- א. How many computers does the principal want to buy?

The budget allocated for buying the equipment was 130,000 shekels.

Therefore, the principal decided to reduce the number of computers ordered by 15% and the number of interactive whiteboards by 10%.

- ב. How much of the equipment budget will be left after the order has been reduced?

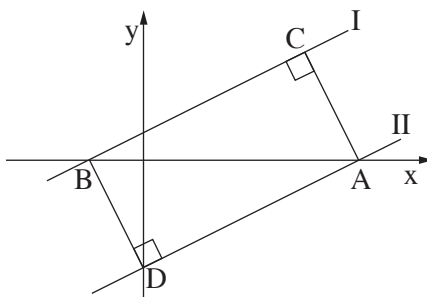
2. Given: two lines, I and II:

$$\text{I. } y = \frac{1}{2}x + 1$$

$$\text{II. } y = \frac{1}{2}x - 4$$

Line I intersects the x-axis at Point B.

Line II intersects the x-axis at Point A (see figure).



- א. Find the coordinates of Point A and the coordinates of Point B.

A line that is perpendicular to Line I is drawn through Point A.

This perpendicular line intersects Line I at Point C (see figure).

- ב. (1) Find the equation of the perpendicular line AC.
(2) Find the coordinates of Point C.

A line that is perpendicular to Line II is drawn through Point B.

This perpendicular line intersects Line II at Point D (see figure).

- ג. What type of a quadrilateral is ACBD? Explain.

- ד. Find the area of the quadrilateral ACBD.

3. Given: a circle whose equation is $(x + 2)^2 + (y - 4)^2 = 20$.

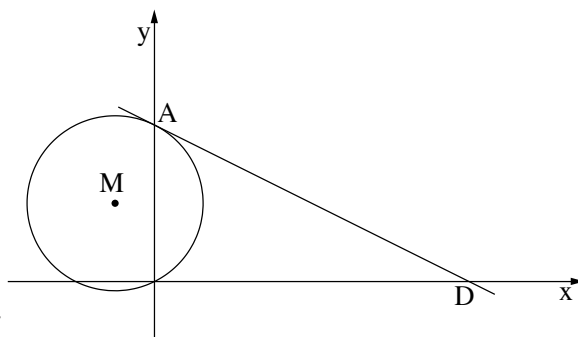
The circle intersects the positive section of the y-axis

at Point A (see figure).

- א. Find the coordinates of Point A.

M is the center of the circle.

The continuation of AM intersects the circle at Point C.



- ב. Find the coordinates of Point C.

A tangent to the circle was drawn through Point A.

- ג. Find the equation of the tangent.

The tangent intersects the x-axis at Point D.

- ד. Find the coordinates of Point D.

Differential and Integral Calculus

4. Given: The function $y = -\frac{1}{2} \cdot x^2 + 2 \cdot \sqrt{x} + 1$.

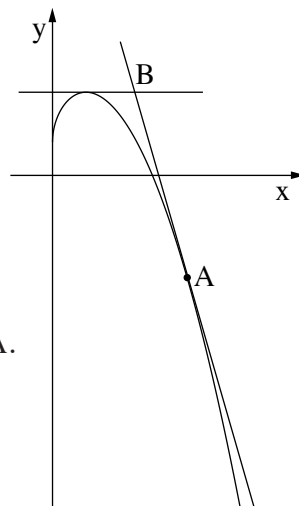
א. What is the domain [תחום ההגדרה] of the function?

A tangent to the graph was drawn through Point A, where $x = 4$ (see figure).

ב. (1) Find the slope of the tangent at Point A.

(2) Find the equation of the tangent at Point A.

ג. Find the coordinates of the maximum point of the function.



At Point B, the line that is a tangent to the graph of the function at Point A meets the line that is a tangent to the graph at its maximum point (see figure).

ד. (1) What is the equation of the line that is a tangent to the maximum point of the function?

(2) Find the coordinates of Point B.

Your answer should be accurate to one decimal place.

5. Given: The derivative function $f'(x) = 3x^2 - 6$.

The line $y = 6x - 14$ is a tangent to the graph of the function $f(x)$ at Point A. Point A lies in the first quadrant.

א. (1) What is the slope of the tangent at Point A?

(2) Find the coordinates of the point of tangency [נקודת ההשקה] A.

ב. Find the function $f(x)$.

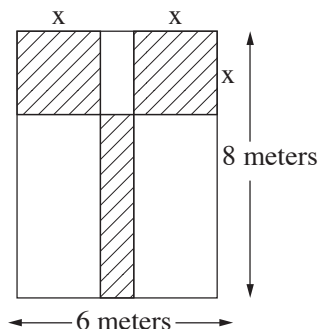
6. Given: a rectangular garden.

The rectangle is 8 meters by 6 meters in size (see figure).

We want to plant grass in the shaded areas of the figure: Two of the areas are identical squares and one area is rectangular, as shown in the figure.

It costs 60 shekels to plant 1 m² of grass.

x denotes the length of a side of each square.



- א. Express the total shaded area in the figure in terms of x .
- ב. What is the value of x for which the area planted with grass will be smallest?
- ג. Find the minimum price for planting the grass.

Good Luck!

All rights reserved by the State of Israel.
 Copying or publication without the consent of the
 Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך