The State of Israel
Ministry of Education
Exam Type:
א. *Bagrut* for high schools
ב. *Bagrut* for external examinees
Exam Date: Summer 2013
Exam Number: 035804, 314
Appendix: Formula sheets for 4 points
English Translation (3)

Mathematics
4 Points — First Questionnaire

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:
This exam has three parts.
Part One: Algebra, Analytical Geometry, Probability – $2\times16\frac{2}{3}$ — $33\frac{1}{3}$ points
Part Two: Planar Geometry and Planar Trigonometry – $2\times16\frac{2}{3}$ — $33\frac{1}{3}$ points
Part Three: Differential and Integral Calculus – $2\times16\frac{2}{3}$ — $33\frac{1}{3}$ points
Total — 100 points

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of the exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/ foreign language-Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of the exam.
(3) For draft paper, use pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of the exam.

Good Luck!

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מדינת ישראל
משרד החינוך
סוג הבחינה:
א. בגרות לבתי ספר על־יסודיים
ב. בגרות לנבחנים אקסטרניים
מועד הבחינה: קיץ תשע"ג, 2013
מספר השאלון: 035804, 314
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה
4 יחידות לימוד-שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים.
פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות
$2\times16\frac{2}{3}$ — $33\frac{1}{3}$ נקודות
פרק שני: גאומטריה וטריגונומטריה במישור
$2\times16\frac{2}{3}$ — $33\frac{1}{3}$ נקודות
פרק שלישי: חשבון דיפרנציאלי ואינטגרלי
$2\times16\frac{2}{3}$ — $33\frac{1}{3}$ נקודות
סה"כ — 100 נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון הניתן לתכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:
(1) אל תעתיק את השאלה; סמן את מספרה בלבד.
(2) התחל כל שאלה בעמוד חדש. רשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. הסבר את כל פעולותיך, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.
(3) לטיוטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים. שימוש בטיוטה אחרת עלול לגרום לפסילת הבחינה.

בהצלחה!

המשך מעבר לדף ◀

Questions

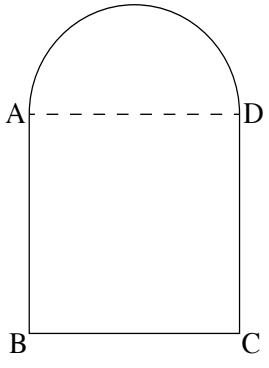
Note! Explain all of your work, including calculations, clearly and in detail. Lack of detail may lower your score or lead to the disqualification of your exam.

Part One — Algebra, Analytical Geometry and Probability ($33\frac{1}{3}$ points)

Answer two of the questions 1-3 (each question — $16\frac{2}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. A window is made up of a semicircle and Square ABCD. The side AD of the square is the diameter of the semicircle, as shown in the figure.



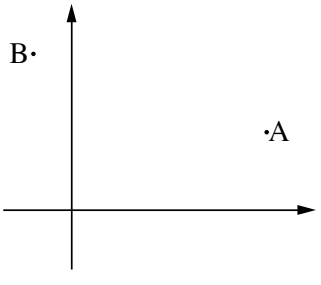
The area of the square is more than $0.2187\ m^2$ greater than the area of the semicircle.

Find the perimeter of the external frame of the window.

In your calculations, use the value $\pi = 3.14$.

2. The following points are given: A (10, 4) and B (–2, 8) (see figure).

Point P is located on the x-axis and is an equal distance from both Point A and Point B.



א. Find the coordinates of Point P.

Points A, B and P are vertices of the quadrilateral ADBP.

Given: $BP \parallel AD$ and $BD \parallel PA$.

ב. Find the coordinates of the vertex D.

ג. Find the length of the radius of the circle that circumscribes [חוסם] Triangle BDA. Explain.

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3. A farmer exports white flowers and red flowers. In the farmer's warehouse:
   $\frac{1}{12}$ of the white flowers are roses and
   $\frac{2}{3}$ of the red flowers are roses.
   25% of the flowers in the warehouse are roses and the rest are lilies.

   א. A flower is chosen at random from among the flowers in the warehouse.
      (1) What is the probability that this flower is red?
      (2) What is the probability that this flower is red, if we know that it is a rose?

   ב. Given: There are 300 red roses in the warehouse.
      What is the total number of flowers in the warehouse?

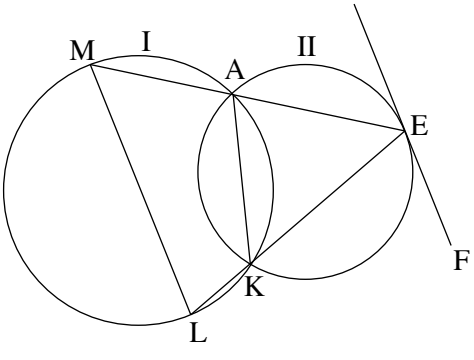
Part Two — Planar Geometry and Planar Trigonometry

($33\frac{1}{3}$ points)

Answer <u>two</u> of the questions 4-6 (each question — $16\frac{2}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

4. Quadrilateral AKLM is inscribed [חסום] within Circle I.
   Circle II passes through Vertices A and K.
   The extensions of the sides MA and LK meet at Point E, which is on Circle II.
   The line FE is a tangent to Circle II at Point E (see figure).



   א. Prove that the line FE is parallel to the chord [מיתר] LM.

   ב. Prove that $\Delta AEK \sim \Delta LEM$.

   ג. Given: AE = 6 cm, KE = 7 cm and KL = 2 cm.
      (1) Calculate the ratio between the area of Triangle AEK and the area of Triangle LEM.
      (2) Calculate the ratio between the area of Triangle AEK and the area of Quadrilateral AKLM.

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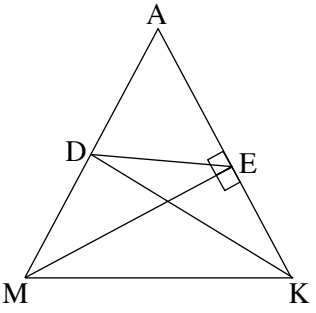
5. In the isosceles triangle [משולש שווה-שוקיים] AMK (AM = AK),
KD is a median [תיכון] to Side AM and
ME is the height to Side AK (see figure).



א. Prove that $\sphericalangle DAE = \sphericalangle DEA$.

Given: $\sphericalangle MAK = 2\alpha$; AM = 2b.

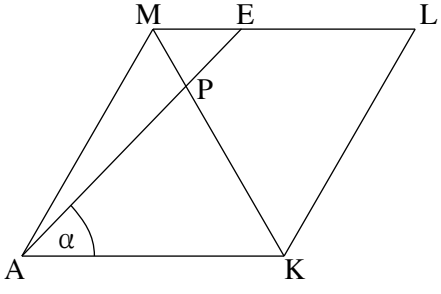
ב. Express the value of the area of
Triangle ADE in terms b and α.

ג. If we also know that MK = 2·DE:

(1) Calculate the value of α.

(2) Prove that $DE \parallel MK$.

6. The rhombus [מעוין] AMLK is given.
Point E is located on Side ML of the rhombus.
The diagonal KM intersects the segment
AE at Point P (see figure).



Given: $\sphericalangle AML = 120^o$, $\sphericalangle EAK = \alpha$ and
the length of a side of the rhombus is a.

א. (1) Find the size of the angle PKA.
Explain.

(2) Express the length of Segment PK
in terms of a and α.

ב. A line that is perpendicular [אנך] to
Side AK is drawn through Point P.
That perpendicular line intersects AK at Point G.
We also know that $\alpha = 46^o$.
Express the length of Segment GL in terms of a.

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Part Three — Differential and Integral Calculus of Polynomials, Rational Functions and Square-Root Functions ($33\frac{1}{3}$ points)

Answer two of the questions 7-9 (each question — $16\frac{2}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

7. The following function is given: $f(x) = x\sqrt{4x} - 6x$.

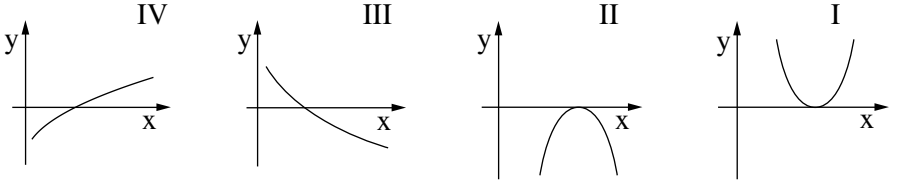
   א. (1) Find the domain [תחום ההגדרה] of the function.

   (2) Find the points at which the graph of the function intersects the axes.

   (3) Find the coordinates of the extrema [נקודות הקיצון] of the function and determine what type of extrema they are.

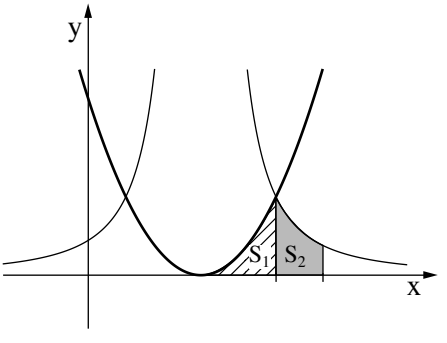
   ב. Sketch a graph of the function.

   ג. Which of the graphs below (I, II, III, IV) could describe the derivative function $f'(x)$ over the interval $1 \le x \le 10$? Explain.



8. The figure on the right shows the graphs of the functions: $f(x) = (x-a)^2$

$$g(x) = \frac{16}{(x-a)^2}$$



a is a parameter greater than 0.

א. Find the asymptotes of the function g(x) that are parallel to the axes. (Express in terms of a, if necessary.)

One of the points at which the graphs of the two functions intersect is the point at which $x = a + 2$.

S_1 is the area bordered by the graph of the function f(x), the x-axis and the line $x = a + 2$ (the area in the figure filled with lines).

S_2 is the area bordered by the graph of the function g(x), the x-axis and the lines $x = a + 2$ and $x = a + 3$ (the area in the figure shaded gray).

ב. Calculate the ratio $\frac{S_1}{S_2}$.

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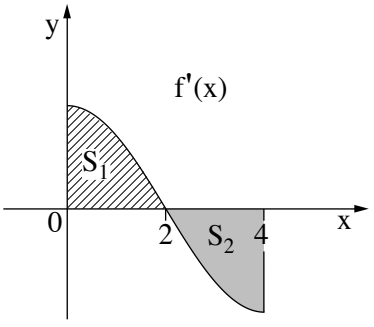
9. The illustration on the right shows the graph of the derivative function $f'(x)$ over the interval $0 \le x \le 4$.



The graph of $f'(x)$ intersects the x-axis at a point at which $x = 2$.

S_1 is the area bordered by the graph of the derivative function $f'(x)$ and the axes (the area in the figure filled with lines).

S_2 is the area bordered by the graph of the derivative function $f'(x)$, the x-axis and the line $x = 4$ (the area in the figure shaded gray).

א. (1) Given: $f(0) = 0$, $S_1 = 4$.
Calculate $f(2)$.

(2) Also given: $S_2 = 4$.
Calculate $f(4)$.

ב. Find the coordinates of the internal extremum [נקודת הקיצון הפנימית] of the function $f(x)$ within the given interval and determine what type of extremum it is. Explain.

ג. Sketch the graph of the function $f(x)$ over the given interval.

Good Luck!

בהצלחה!