

Mathematics

5 Points — First Questionnaire

Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra and Probability

$$- 2 \times 16 \frac{2}{3} - 33 \frac{1}{3} \text{ points}$$

Part Two: Planar Geometry and Planar Trigonometry

$$- 2 \times 16 \frac{2}{3} - 33 \frac{1}{3} \text{ points}$$

Part Three: Differential and Integral Calculus

$$- 2 \times 16 \frac{2}{3} - 33 \frac{1}{3} \text{ points}$$

Total — 100 points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of the exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language / foreign language-Hebrew dictionary

ד. Special instructions:

(1) Do not copy the question; only indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of the exam.

(3) For draft paper, use the pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of the exam.

מתמטיקה

5 יחידות לימוד — שאלון ראשון

הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה והסתברות

$$- 2 \times 16 \frac{2}{3} - 33 \frac{1}{3} \text{ נקודות}$$

פרק שני: גאומטריה וטריגונומטריה במישור

$$- 2 \times 16 \frac{2}{3} - 33 \frac{1}{3} \text{ נקודות}$$

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

$$- 2 \times 16 \frac{2}{3} - 33 \frac{1}{3} \text{ נקודות}$$

סה"כ — 100 נקודות

ה. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש

באפשרויות התכנות במחשבון הניתן

לתכנות. שימוש במחשבון גרפי או

באפשרויות התכנות במחשבון עלול לגרום

לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ו. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום

במחברת את שלבי הפתרון, גם כאשר

החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או

לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה או

בדפים שקיבלת מהמשגיחים.

שימוש בטייטה אחרת עלול לגרום לפסילת

הבחינה.

Good Luck!

/Continued on the next page/

בהצלחה!

/המשך מעבר לדף/

Questions

Note! Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of the exam.

Part One — Algebra and Probability ($33\frac{1}{3}$ points)

Answer two of the questions 1-3 (each question – $16\frac{2}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

1. A motorcycle rider left Location A at the same time that a bicycle rider left Location B. They rode toward each other and met each other.
The motorcycle rider arrived at Location B $\frac{1}{4}$ of an hour after they met.
The bicycle rider arrived at Location A 4 hours after they met.
(Each rider traveled at a constant speed.)
 - א. Find the ratio of the speed of the motorcycle rider to the speed of the bicycle rider.
 - ב. We know that the distance between A and B is greater than 90 km.
Find the range of numbers within which the speed of each rider is found.
(The speed of the motorcycle rider does not exceed 120 km/h.)
2. Given: a geometric sequence $a_1, a_2, a_3, \dots, a_n$
 - א. Using induction or any other method, prove that for every natural n :

$$a_1^2 \cdot a_2^2 \cdot a_3^2 \cdot \dots \cdot a_n^2 = (a_1 \cdot a_n)^n$$
 - ב. Given: $a_1^4 \cdot a_6^4 = 1,048,576$
 - (1) Calculate the product $a_1^2 \cdot a_2^2 \cdot a_3^2 \cdot \dots \cdot a_6^2$.
 - (2) We also know that $a_1 = 1$.
Calculate the product $a_1^2 \cdot a_2^2 \cdot a_3^2 \cdot \dots \cdot a_7^2$.

Note: You may leave exponents in your answers.

3. A survey was conducted among a large number of students (boys and girls).
 Half of the students who participated in the survey are boys.
 The survey found that the number of girls who are bothered by noise is
 3 times greater than the number of boys who are bothered by noise.
 It also found that 5% of the boys are bothered by noise.
- א. One of the survey participants was randomly selected.
 We know that he/she is bothered by noise.
 What is the probability that the selected participant is a girl?
- ב. 5 students were randomly selected from among the participants in the
 survey. We know that no more than 2 of the 5 students who were
 randomly chosen are bothered by noise. What is the probability that
 exactly one of the selected students is bothered by noise?

Part Two — Planar Geometry and Planar Trigonometry

($33\frac{1}{3}$ points)

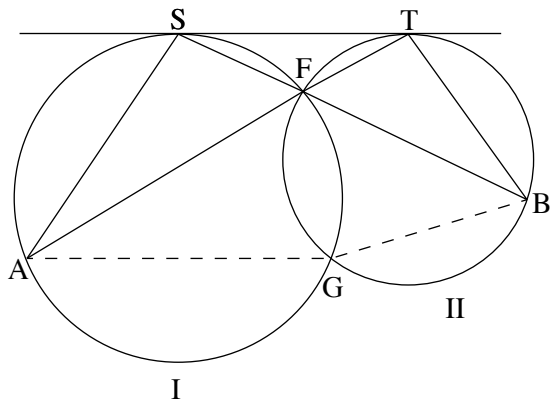
Answer two of the questions 4-6 (each question – $16\frac{2}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your
 answer booklet will be graded.

4. Two circles, I and II, intersect at
 Points G and F.

The line ST is tangent to Circle I
 at Point S and is tangent to
 Circle II at Point T.

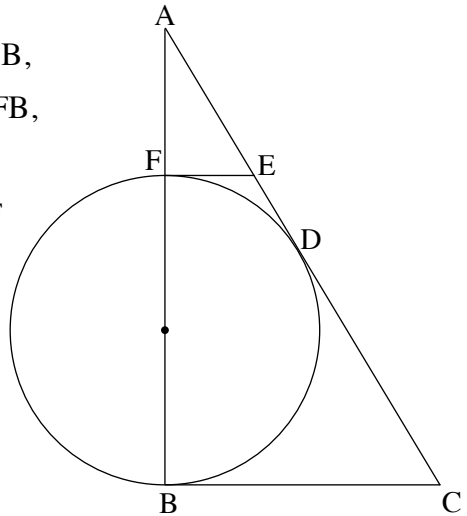
The extension of SF intersects
 Circle II at Point B and
 the extension of TF intersects
 Circle I at Point A (see figure).



- א. Prove that $\frac{ST}{AS} = \frac{TB}{ST}$.
- ב. (1) Prove that $\angle AGF = \angle SFA + \angle SAF$.
 (2) Prove that if Points A, G and B are
 located on one line, then $\angle SFA = 60^\circ$.

5. Sides CA and CB of Triangle ABC are tangents to the circle at Points D and B, respectively. The diameter of the circle, FB, lies on Side AB.

Point E is located on Side AC, so that EF is tangent to the circle (see figure).



- א. Can the quadrilateral FEDB be circumscribed [בר חסימה] by the circle? Explain.

Given: $EC = 15 \text{ cm}$, $AE = 5 \text{ cm}$.

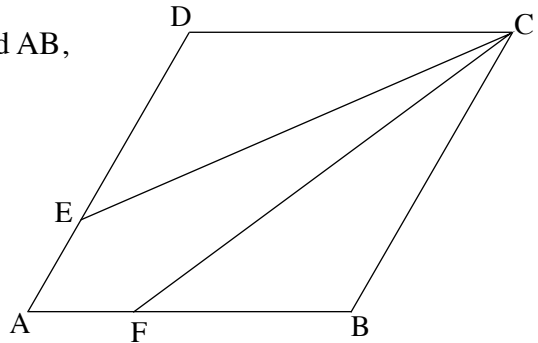
- ב. Prove that $CB + EF = ED + CD$.
ג. Calculate the length of EF. Explain.
ד. Calculate the size of the angles in Triangle FDB.

6. Given: Rhombus ABCD

E and F are points on Sides AD and AB, respectively, so that $AE = AF$ and $FB = 2AF$.

Given: $\angle DCB = 60^\circ$

- א. Find the size of Angle FCB.
ב. Given: The length of the diagonal AC is b.



Express the perimeter of Quadrilateral AECF in terms of b.

Part Three — Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions and Trigonometric Functions $(33\frac{1}{3} \text{ points})$

Answer two of the questions 7-9 (each question – $16\frac{2}{3}$ points).

Note! If you answer more than two questions, only the first two answers in your answer booklet will be graded.

7. Given: the function $f(x) = \cos^3(3x - \pi)$, defined for every x

א. Over the interval $0 \leq x \leq \frac{2\pi}{3}$, find:

- (1) the points at which the graph of the function intersects the axes.
- (2) the coordinates of the extrema [נקודות הקיצון] of the function and determine what type of extrema they are.

ב. (1) Prove that the function is even.

(2) Sketch a graph of the function over the interval $-\frac{2\pi}{3} \leq x \leq \frac{2\pi}{3}$.

ג. Write out the equations of the lines that are tangents to the graph of the function within the interval $-\frac{2\pi}{3} \leq x \leq \frac{2\pi}{3}$ and are perpendicular to the y-axis.

8. Given: the function $f(x) = \frac{x+1}{\sqrt{x^2-9}}$

א. Find:

- (1) the domain [תחום ההגדרה] of the function.
- (2) the points at which the graph of the function intersects the axes (if such points exist).
- (3) the asymptotes of the function that are parallel to the axes.
- (4) the intervals over which the function increases and decreases.

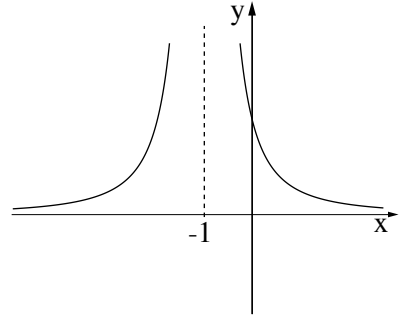
ב. Sketch a graph of the function.

ג. Find the sign of the definite integral $\int_k^t f'(x) dx$ ($k < t$),

if we know that k and t are each greater than 3. Explain.

9. The function $f(x)$ is a rational function [פונקציית מנה] that is defined for $x \neq -1$.

The figure shows the graph of the derivative function $f'(x)$.



- א. Find the intervals of convexity $\cup$ and the intervals of concavity $\cap$ of the function. Explain.

- ב. We know that the function $f(x)$ has only two asymptotes: $x = -1$ and $y = 1$.

The graph of the function $f(x)$ intersects the y-axis at the point at which $y = -1$.

Sketch a graph of the function $f(x)$ based on your answer for Item א and the information in Item ב.

- ג. Also given: $f(x) = \frac{ax + b}{cx + d}$

a, b, c and d are parameters that are each different from 0.

- (1) Express b, c and d in terms of a .
- (2) Calculate the size of the area that is bordered by the graph of the derivative function $f'(x)$, the line $x = 1$ and the axes.

Good Luck!

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 Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך