

## Mathematics

### 4 Points — First Questionnaire

#### Instructions for examinees

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts.

Part One: Algebra, Analytical Geometry,

$$\text{Probability} - 2 \times 16\frac{2}{3} - 33\frac{1}{3} \text{ points}$$

Part Two: Planar Geometry and Planar Trigonometry

$$- 2 \times 16\frac{2}{3} - 33\frac{1}{3} \text{ points}$$

Part Three: Differential and Integral Calculus

$$- 2 \times 16\frac{2}{3} - 33\frac{1}{3} \text{ points}$$

$$\text{Total} - 100 \text{ points}$$

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of the exam.

(2) Formula sheets (attached)

(3) A Hebrew-foreign language /

foreign language-Hebrew dictionary

ד. Special instructions:

(1) Do not copy the question; only indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of the exam.

(3) For draft paper, use the pages in the answer booklet or paper you receive from the proctors. Use of other draft paper may lead to the disqualification of the exam.

**Good Luck!**

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## מתמטיקה

### 4 יחידות לימוד — שאלון ראשון

#### הוראות לנבחן

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים.

פרק ראשון: אלגברה, גאומטריה אנליטית, הסתברות

$$- 2 \times 16\frac{2}{3} - 33\frac{1}{3} \text{ נקודות}$$

פרק שני: גאומטריה וטריגונומטריה במישור

$$- 2 \times 16\frac{2}{3} - 33\frac{1}{3} \text{ נקודות}$$

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי

$$- 2 \times 16\frac{2}{3} - 33\frac{1}{3} \text{ נקודות}$$

$$\text{סה"כ} - 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש

באפשרויות התכנות במחשבון הניתן

לתכנות. שימוש במחשבון גרפי או

באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש. רשום

במחברת את שלבי הפתרון, גם כאשר

החישובים מתבצעים בעזרת מחשבון.

הסבר את כל פעולותיך, כולל חישובים,

בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

(3) לטייטה יש להשתמש במחברת הבחינה או בדפים שקיבלת מהמשגיחים.

שימוש בטייטה אחרת עלול לגרום לפסילת הבחינה.

**בהצלחה!**

/המשך מעבר לדף/

## Questions

**Note! Explain all of your work, including calculations, clearly and in detail.**  
**Lack of detail may lower your score or lead to the disqualification of the exam.**

### Part One —

#### Algebra, Analytical Geometry and Probability ( $33\frac{1}{3}$ points)

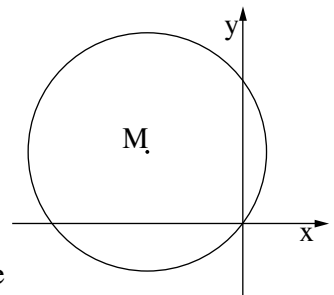
Answer two of the questions 1-3 (each question —  $16\frac{2}{3}$  points.)

**Note!** If you answer more than two questions, only the first two answers in your answer booklet will be graded.

- For customers of a certain telephone company, the cost per minute for evening phone calls is 40% less than the cost per minute for daytime phone calls. In order to encourage customers to talk on the phone in the evening, the company lowered the cost per minute for evening calls by 18%.  
(The cost per minute for daytime phone calls was not changed.)  
After the price decrease, Elad spoke for 150 minutes during the day and 300 minutes in the evening and paid 44.64 shekels.  
Find the cost per minute (in agorot) of daytime phone calls and the cost per minute for evening phone calls before the price was lowered.

- Given: a circle whose equation is  $(x - a)^2 + (y - 3)^2 = 25$   
 $a$  is a parameter.

The circle passes through the origin of the axes  
and the center  $M$  is located in the second quadrant  
(see figure).



- Find the value of  $a$ .
- Find the coordinates of the points on the circle that have a  $y$ -coordinate that is 2 greater than their  $x$ -coordinate.
- A tangent to the circle is drawn through each of the points that you found for Item ב.

Find the equations of these tangents.

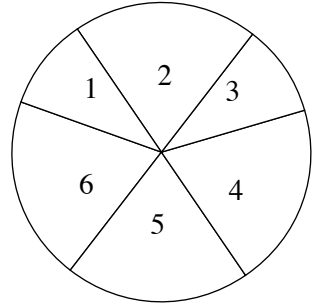
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3. A balanced game wheel is divided into six sectors.

The numbers 1 and 3 are written on 2 of the sectors, each of which constitutes  $\frac{1}{10}$  of the circle.

The numbers 2, 4, 5 and 6 are written on 4 sectors, each of which constitutes  $\frac{1}{5}$  of the circle, as shown

in the figure. When the wheel is spun, it stops on one of the numbered sectors (not on one of the lines between the sectors).



א. The wheel is spun once.

What is the probability that the wheel will stop on an even number?

The wheel is spun 5 times.

ב. (1) What is the probability that the wheel will stop on an even number no more than 2 times?

(2) We know that the wheel stopped on an even number no more than 2 times. What is the probability that the wheel stopped on an even number exactly 2 times?

ג. What is the probability that the wheel stopped on an even number on the first and last spins only?

## Part Two — Planar Geometry and Planar Trigonometry

( $33\frac{1}{3}$  points)

Answer two of the questions 4-6 (each question —  $16\frac{2}{3}$  points).

**Note!** If you answer more than two questions, only the first two answers in your answer booklet will be graded.

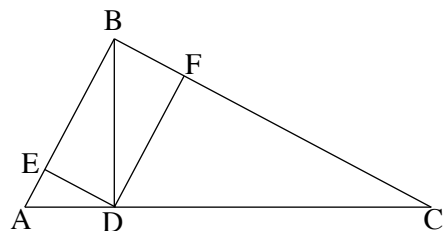
4. Given: a right triangle ( $\angle ABC = 90^\circ$ )

BD is the height to the hypotenuse AC.

F is a point on BC, such that  $DF \perp BC$ .

E is a point on BA, such that  $DE \perp BA$

(see figure).



א. Prove that EF and BD are equal to

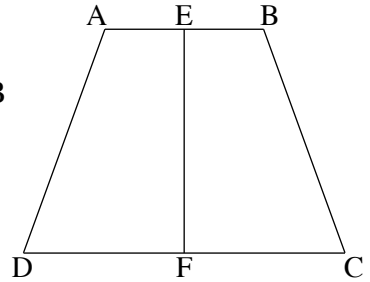
each other and that they intersect each other at their respective midpoints.

ב. Prove that  $ED^2 = DF \cdot AE$ .

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5. Given: the isosceles trapezoid [טרפז שווה-שוקיים]  
ABCD ( $AB \parallel DC$ )

Points E and F are the midpoints of the bases AB and DC, respectively (see figure).



- א. (1) Prove that  $ED = EC$ .  
(2) Prove that  $EF \perp DC$ .

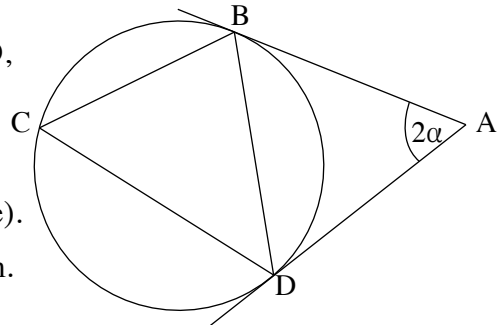
ב. Given:  $AB = 4$  cm

$$BC = 6 \text{ cm}$$

$$\angle EBC = 110^\circ$$

Find the size of the angle ECB.

6. Two tangents to the circle, AB and AD, were drawn from Point A.  
Point C is located on the circle, outside of the triangle ABD (see figure).  
Given: the radius of the circle is 10 cm.



$$\angle BAD = 2\alpha$$

- א. (1) Prove that  $\angle BCD = 90^\circ - \alpha$ .  
(2) Express the length of AB in terms of  $\alpha$ .
- ב. We also know that  $\alpha = 30^\circ$  and  $\angle CBD = 70^\circ$ .  
Calculate the length of AC.

## Part Three — Differential and Integral Calculus of Polynomials, Rational Functions and Root Functions

( $33\frac{1}{3}$  points)

Answer two of the questions 7-9 (each question —  $16\frac{2}{3}$  points).

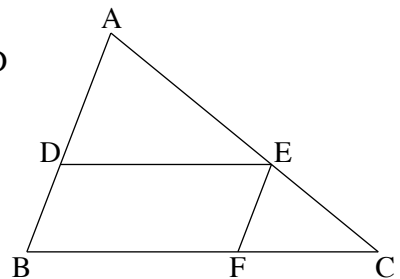
**Note!** If you answer more than two questions, only the first two answers in your answer booklet will be graded.

7. Given: the function  $f(x) = -x^2\sqrt{x+5}$

- א. Find the domain [תחום ההגדרה] of the function.
- ב. Find the points at which the graph of the function intersects the axes.
- ג. Are there values of  $x$  for which  $f(x) > 0$ ? Explain.
- ד. Find the coordinates of the extrema [נקודות הקיצון] of the graph of the function and determine what type of extrema they are.
- ה. Sketch the graph of the function.
- ו. How many solutions are there for the equation:  $-14 = -x^2\sqrt{x+5}$ ? Explain.

8. Given: Parallelogram DEFB, in which the lengths of the sides are  $BD = 40$  cm and  $DE = 90$  cm.

Point A is located on the extension of Side BD and Point C is located on the extension of Side BF, such that the line AC passes through Vertex E (see figure).



א. Indicated:  $AD = x$ .

Using the properties of similar triangles,  
express the length of the segment FC in terms of  $x$ .

- ב. Find the value of  $x$  for which the sum of the sides AB and BC is smallest.
- ג. Find the smallest value of the sum of Sides AB and BC.

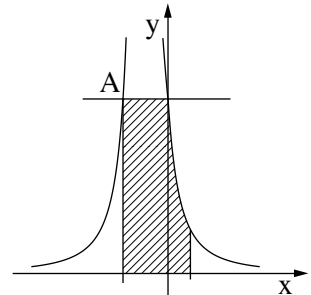
9. The figure below shows a graph of the function  $f(x) = \frac{4}{(2x+1)^2}$ .

א. Find the domain [תחום ההגדרה] of the function.

ב. Find the asymptotes of the function that are perpendicular to the axes.

ג. A line that is parallel to the x-axis was drawn through the point at which the graph of the function intersects the y-axis.

The line intersects the graph of the function at another point, A (see figure).



(1) Find the coordinates of Point A.

(2) A line that is perpendicular to the x-axis was drawn through Point A.

Find the size of the area that is bordered by the perpendicular line, the parallel line, the graph of the function, the line  $x = \frac{1}{2}$  and the x-axis (the shaded area in the figure).

**Good Luck!**

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**בהצלחה!**

זכות היוצרים שמורה למדינת ישראל  
 אין להעתיק או לפרסם אלא ברשות משרד החינוך